**Pigeon-hole Principle for Valid Node Degrees**

Suppose we want to know if it’s possible to construct a *simple* undirected graph with degree set {d1, d2, d3, … , dn}, where d1 ≥ d2 ≥ d3, … ≥ dn.

Necessary Conditions:

1. Let D = dk, the sum of the degrees. D must be even. (D = (2\*#edges) which is even).
2. di <= n-1 for all i. (If di >= n then, by the pigeonhole principle, node i must connect to some other node at least twice, which means the graph not simple).
3. di <= D – di. (No node can have degree greater than what is required by the rest of the graph).

Condition c) is checked for individual nodes. If the condition holds more generally, i.e., for all subsets of {d1, d2, d3, …, dn}, then the conditions would be *sufficient* to prove that the desired simple graph exists.

Since the degree values are listed in descending order, we need only look at the subsets: S1 = {d1}, S2 = {d1, d2}, ..., Sk = {d1, d2, …, dk}, …, Sn = {d1, d2, d3, …., dn}. So, let Dk = the sum of the degrees within Sk. The more general *necessary and sufficient conditions* for the graph to exist are a), b), and:

c’) Dk - (k)(k-1) <= (D – Dk) for all k=1…n. (Sk can host as much as a complete subgraph having total degree (k)(k-1), so the minimum number of edges leaving Sk cannot be greater than the degree requirements of the rest of the graph).

If any of these conditions is violated then, by the pigeonhole principle it is impossible to construct the graph without repeating a link. Otherwise, the simple graph can be built in polynomial time.

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